Energy Efficient Microgrid Management using Model Predictive Control

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Abstract— Microgrids are subsystems of the distribution grid which comprises small generation capacities, storage devices and controllable loads, operating as a single controllable system that can operate either connected or isolated from the utility grid. In this paper we present a preliminary study on applying a Model Predictive Control (MPC) approach to the problem of efficiently optimizing microgrid operations while satisfying a time-varying request and operation constraints. The overall problem is formulated using Mixed-Integer Linear Programming (MILP), which can be solved in an efficient way by using commercial solvers without resorting to complex heuristics or decompositions techniques. Then the MILP formulation leads to significant improvements in solution quality and computational burden. A case study of a typical microgrid is employed to assess the performance of the on-line optimization-based control strategy: simulation results show the feasibility and the effectiveness of the proposed approach.

I. INTRODUCTION

The need to satisfy in sustainable ways the increasing energy demand requires *active* energy distribution networks, i.e. distribution networks with the possibility of bidirectional power flows controlling a combination of Distributed Energy Resources (DERs), such as distributed generators and renewable energy devices. Hence, new energy management systems are needed, able to optimally control the distributed generation in the distribution network. In this scenario, the microgrid concept is a promising approach. It is an integrated energy system consisting of interconnected loads and DERs which can operate in parallel with the grid or in an intentional island mode, e.g. see [1]. A typical microgrid comprises: storage units; Distributed Generators (DGs), which are dispatchable units; Renewable Energy Resources (RESs), which are noncontrollable devices; and controllable loads, which can be curtailed (shed) when it is more convenient. In addition a microgrid can purchase and sell power to and from its energy suppliers. The optimization of the microgrid operations is extremely important in order to cost-efficiently manage its energy resources [1]. In this paper we tackle the optimal operation planning of a microgrid. This problem aims at minimizing the overall microgrid operating costs to meet the predicted load demand of a certain period (typically one day) while satisfying complex operational constraints, such as the energy balance and controllable generators minimum operation time and minimum stop time. A complete formulation of microgrid optimal operation planning problem includes modeling of storage, demand side policies for controllable loads (Demand Side Management, DSM),

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power exchange with the utility grid. Moreover, microgrid modeling needs both continuous (such as storage output) and discrete (such as on/off states of DGs and DSM-controlled loads) decision variables. Due to the problem complexity and because of the large economic benefits that could result from its improved solution, considerable attention is being devoted to development of better optimization algorithms and suitable modeling frameworks. Studies have suggested that microgrids can achieve high performance through: (*i*) advanced control algorithms accounting for system uncertainty and based on predicted future conditions; (*ii*) deployment of demand response; (*iii*) optimal use of storage devices in order to compensate the physical imbalances; (*iv*) applying optimal instead of heuristic-based approaches, e.g. see [2], [3] and the references therein. The proposed approaches are typically either computationally intensive and not suitable for real-time applications, or can produce suboptimal solutions, e.g. see [3]–[5].

In this paper we present a control-oriented approach to microgrid modeling and optimization and propose the use of Model Predictive Control (MPC) in combination with Mixed Integer Linear Programming (MILP) [6]. By doing so, the optimization problems can be solved very efficiently by standard algorithms and the feedback mechanism can take into account the uncertainty in microgrid operations associated with (*i*) the RES power outputs; (*ii*) the time-varying load; (*iii*) time-varying energy prices. To the best of our knowledge, very little work can be found in the literature that addresses Model Predictive Control for optimal dispatch in power systems and in microgrid in particular. The authors in [7] propose a look-ahead model predictive control algorithm to solve the economic dispatch problem with large presence of intermittent resources. However, many microgrid key features such as minimum up and down times, demand side programs, storages and on/off generators status are not considered. In [8] a model predictive controller is applied to controlling the energy flows inside a household system equipped with a 'micro' combined heat and power unit. In addition, the household can buy and sell electricity from/to the energy supplier; heat and electricity can be stored in specific storage devices.

Our contributions are: (*i*) the development of a model of the overall microgrid system adopting a formalized modeling approach, which is suitable to be used in on-line optimization schemes; (*ii*) the development of a MPC scheme for minimizing the microgrid running costs; (*iii*) the presentation of preliminary simulation results showing the effectiveness of the proposed optimization routine.

The paper is organized as follows: the microgrid system is

described and the microgrid modeling approach is outlined in Section II; the operations optimization is then described in Section III; finally, in Section IV some simulation results are discussed.

A. Nomenclature

The forecasts, the parameters and the decision variables used in the proposed formulation are described respectively in Tables I, II and III, where, for simplicity, we omit the subscript 'i' when referring to the ith unit.

TABLE I
PARAMETERS

Parameters	Description
N_g, N_l, N_c	number respectively of DG units, critical loads and controllable loads
$C^{DG}(P)$	fuel consumption cost curve of a DG unit
a_1, a_2, a_3	cost coefficients of $C^{DG}(P)$ [€/(kWh)^2, €/kWh, €]
OM	operating and maintenance cost of a DG unit [€/kWh]
$R_{\max}$	ramp up limit of a DG unit [kW/h]
T^{up}, T^{down}	minimum up and down time of a DG unit [h]
x^{sb}	storage 'physiological' energy loss [kWh]
$x^b_{\min}$, $x^b_{\max}$	minimum, maximum energy level of the storage unit [kWh]
$C^b_{\max}$	storage power limit [kW]
T^g	maximum interconnection power flow limit (at the point of common coupling) [kW]
$P_{\min}$, $P_{\max}$	minimum, maximum power level of a DG unit [kW]
η^c, η^d	storage charging, discharging "efficiencies"
$\beta_{\min}$, $\beta_{\max}$	minimum, maximum allowed curtailment of a controllable load
c^{SU}, c^{SD}	start-up, start-down costs of a DG unit [€]
D^c	preferred power level of a controllable load [kW]
ρ_c	penalty weight on curtailments

The fuel consumption cost for a DG unit is traditionally assumed to be a quadratic function of the form $C^{DG}(P) = a_1P^2 + a_2P + a_3$.

TABLE II
FORECASTS

Forecasts	Description
P^{res}	sum of power production from RES [kW]
D	power level required from a critical load [kW]
c^P, c^S	purchasing, selling energy prices [€/kWh]

TABLE III
DECISION AND LOGICAL VARIABLES

Variables	Description
δ	off(0)/on(1) state of a DG unit
δ^b	discharging(0)/charging(1) mode of the storage unit
δ^g	exporting(0)/importing(1) mode to/from the utility grid
P	power level of a DG unit [kW]
P^b	power exchanged (positive for charging) with the storage unit [kW]
P^g	importing(positive)/exporting(negative) power level from/to the utility grid [kW]
x^b	stored energy level [kWh]
β	curtailed power percentage

II. SYSTEM DESCRIPTION AND MODELING

Here we briefly describe the key features of the microgrid architecture considered in this paper and associate a possible modeling set up with the goal of maintaining the problem tractable and suitable for real-time computation. When the microgrid is in the grid-connected mode, it can purchase and sell electricity from/to the utility grid. The microgrid produces the electricity using controllable distributed generators and renewable energy resources, and electricity can be stored in a storage device. The energy demand comes from both critical and controllable loads. The optimal use of the storage unit and the controllable loads can help to keep the energy balance, in particular during the islanded mode. The microgrid system comprises continuous time-driven dynamics of the energy flows and storage units, and event-driven on/off controllers.

We point out what follows:

- in a hierarchy of controllers we aim at a high level optimization of microgrid operations; voltage stability, power quality, and frequency are supposed to be controlled at the lower control level;
- the system is considered to be in steady state;
- the microgrid central controller has full information and knowledge of the managed network: it integrates load and generation forecasting tools, and knows the existing generation capacity, storage capacity, network constraints, market energy prices, bilateral contracts;
- heat recovery capabilities and reactive power are not considered in the microgrid modeling and problem formulation to limit its complexity. Yet we are aware of their importance and their incorporation into the proposed control framework is under current study;
- due to constant sampling time $\Delta T = t_{k+1} - t_k$, there exists a constant ratio between energy and power at each interval.

A. Storage Dynamics

We consider the following discrete time model of a storage unit:

$$x^b(k+1) = x^b(k) + \eta P^b(k) - x^{sb}, \qquad (1)$$

where

$$\eta = \begin{cases} \eta^c, & \text{if } P^b(k) > 0 \text{ (charging mode)} \\ \eta^d, & \text{otherwise (discharging mode)}. \end{cases} \qquad (2)$$

We denote by $x^b(k)$ the level of the energy stored at time k (divided by ΔT) and by $P^b(k)$ the power exchanged with the storing device at time k. The charging and discharging 'efficiencies' account for the losses and x_{sb} denotes a constant stored energy degradation in the sampling interval. If the power exchanged at time k, $P^b(k)$, is greater than zero, this will be charging the storage device, otherwise the storage device will be discharged.

By using the standard approach described in [9], we introduce a binary variable $\delta^b(k)$ and an auxiliary variable

$z^b(k) = \delta^b(k)P^b(k)$ to model the logical conditions provided in Section 1 such as:

$$P^b(k) > 0 \iff \delta^b(k) = 1$$

and

$$x^b(k+1) = \begin{cases} x^b(k) + \eta^c P^b(k) - x^{sb}, & \text{if } \delta^b(k) = 1, \\ x^b(k) + \eta^d P^b(k) - x^{sb}, & \text{otherwise.} \end{cases}$$

Then we express the 'if ... then' conditions as mixed integer linear inequalities. By collecting such inequalities we can rewrite the storage dynamics and the corresponding constraints in the following compact form (the interested reader is referred to [9] for guiding details):

$$
\begin{aligned}
x^b(k+1) &= x^b(k) + (\eta^c - \eta^d)z^b(k) + \eta^d P^b(k) - x^{sb}, \\
\text{subject to} \quad & \mathbf{E_1}^b \delta^b(k) + \mathbf{E_2}^b z^b(k) \leq \mathbf{E_3}^b P^b(k) + \mathbf{E_4}^b,
\end{aligned}
\tag{3}
$$

where the column vectors $\mathbf{E_1}^b, \mathbf{E_2}^b, \mathbf{E_3}^b, \mathbf{E_4}^b$ are provided in the Appendix.

The balance between energy production and consumption must be met at each time k, so the following equality constraint is imposed:

$$
\begin{aligned}
P^b(k) &= \textstyle\sum_{i=1}^{N_g} P_i(k) + P^{\text{res}}(k) + P^g(k) \\
&\quad - \textstyle\sum_{j=1}^{N_l} D_j(k) - \textstyle\sum_{h=1}^{N_c}[1 - \beta_h(k)]D_h^c(k).
\end{aligned}
\tag{4}
$$

If we collect all the decision variables in the vector $u(k)$ and all the known disturbances (obtained by forecasts) in the vector $\hat{w}(k)$, we can express the storage level as an affine function by substituting $P^b(k)$ in (3) as follows:

$$
\begin{aligned}
x^b(k+1) &= x^b(k) + (\eta^c - \eta^d)z^b(k) \\
&\quad + \eta^d\left[\mathbf{F}'(k)u(k) + \mathbf{f}'(k)\hat{w}(k)\right] - x^{sb}
\end{aligned}
\tag{5}
$$

with

$$
u(k) = \begin{bmatrix} \mathbf{P}'(k) & P^g(k) & \beta'(k) & \delta'(k) \end{bmatrix}' \in \mathbb{R}^{N_u} \times \{0,1\}^{N_g},
$$

$$
\hat{w}(k) = \begin{bmatrix} P^{\text{res}}(k) & \mathbf{D}'(k) & \mathbf{D^c}'(k) \end{bmatrix}' \in \mathbb{R}^{N_w},
$$

where $N_u = N_g + 1 + N_c$, $N_w = 1 + N_l + N_c$; $\mathbf{P}(k)$, $\delta(k)$, $\mathbf{D}(k)$, $\mathbf{D^c}(k)$ and $\beta(k)$ are column vectors containing, respectively, all the power levels, the generators off/on states, the critical demands, the controllable preferred power levels and the curtailments. We remark that the vector $u(k)$ collects both the continuous-value and the binary control inputs, and the vector $\hat{w}(k)$ collects all the known disturbances (obtained by forecasts). The vectors $\mathbf{F}(k)'$ and $\mathbf{f}(k)'$ are provided in the Appendix.

B. Interaction with the utility grid

When grid-connected, the microgrid can sell and purchase energy from/to the utility grid. By following the same procedure outlined above, we introduce a binary variable $\delta^g(k)$ and an auxiliary variable $C^g(k)$ to model the possibility either to purchase or to sell energy from/to the utility grid. For the new variables, the following logical statements must hold:

$$P^g(k) > 0 \iff \delta^g(k) = 1$$

and

$$
C^g(k) = \begin{cases} c^P(k)P^g(k) & \text{if } \delta^g(k) = 1, \\ c^S(k)P^g(k) & \text{otherwise.} \end{cases}
$$

Again, we express the 'if ... then' conditions as mixed integer linear inequalities. Then, the purchasing/selling microgrid behavior can be expressed by the following mixed integer linear inequalities in a compact form:

$$\mathbf{E_1}^g \delta^g(k) + \mathbf{E_2}^g C^g(k) \leq \mathbf{E_3}^g(k)P^g(k) + \mathbf{E_4}^g. \tag{6}$$

The column vectors $\mathbf{E_1}^g, \mathbf{E_2}^g, \mathbf{E_3}^g(k), \mathbf{E_4}^g$ are provided in the Appendix. The matrix $\mathbf{E_3}^g(k)$ is generally time-varying due to the time varying energy prices. We recall that the interaction with the utility grid is allowed only when the microgrid is in the grid-connected mode.

C. Generator operating conditions

The operating constraints, at each sampling time k, on the minimum amount of time for which a controllable generation unit must be kept on/off (minimum up/down times) can be expressed by the following mixed integer linear inequalities without resorting to any additional variable:

$$
\begin{aligned}
\delta_i(k) &\geq \delta_i(k - \tau_{\text{up}} - 1) - \delta_i(k - \tau_{\text{up}} - 2), \\
1 - \delta_i(k) &\geq \delta_i(k - \tau_{\text{down}} - 2) - \delta_i(k - \tau_{\text{down}} - 1),
\end{aligned}
\tag{7}
$$

with $i = 1, \ldots, N_g$, $\tau_{\text{up}} = 0, \ldots, \min(T_i^{\text{up}} - 1, k - T_i^{\text{up}} + 2)$ and $\tau_{\text{down}} = 0, \ldots, \min(T_i^{\text{up}} - 1, k - T_i^{\text{up}} + 2)$.

We also model the DG unit start up and shut down behavior in order to account for the corresponding costs. For this reason, two auxiliary variables, $SU_i(k)$ and $SD_i(k)$ are introduced, representing respectively the start up and the shut down cost for the ith DG generation unit at time k. These auxiliary variables must satisfy the following mixed integer linear constraints:

$$
\begin{aligned}
SU_i(k) &\geq c_i^{SU}(k)[\delta_i(k) - \delta_i(k-1)], \\
SD_i(k) &\geq c_i^{SD}(k)[\delta_i(k-1) - \delta_i(k)], \\
SU_i(k) &\geq 0, \\
SD_i(k) &\geq 0,
\end{aligned}
\tag{8}
$$

with $i = 1, \ldots, N_g$.

D. Loads

We consider two types of loads:

- *critical loads*, i.e. demand levels related to essential processes that must be always met;
- *controllable loads*, i.e. loads that can be reduced or shed during supply constraints or emergency situations (e.g., standby devices, day-time lighting).

In demand response programs the customers specify level of curtailment of the controllable loads. The controllable loads have a preferred level, but their magnitude is flexible so that the demand level can be lowered when it is convenient or necessary (e.g., in islanded mode). This leads to users' discomfort, hence a certain cost is associated with the load curtailment/shedding (a penalty for the microgrid).

We define a continuous-valued variable, $0 \leq \beta_c(k) \leq 1$, associated to each controllable load c and to each sampling

time k. This variable represents the percentage of preferred power level to be curtailed at time k in order to keep the microgrid operations feasible (e.g., in islanded mode) or more economically convenient. If no curtailment is allowed at a certain time $\hat{k}$, an equality constraint can be set, $\beta_c(\hat{k}) = 0$.

III. PROBLEM FORMULATION

In this section we define the microgrid optimization problem. At every time step, the microgrid controller must take high level decisions about:

- when should each generation unit be started and stopped (Unit Commitment);
- how much should each unit generate to meet this load at minimum cost (Economic Dispatch);
- when should the storage device be charged or discharged;
- when and how much energy should be purchased or sold to the utility grid (when the microgrid is in the grid-connected mode);
- curtailment schedule (which controllable loads must be shed/curtailed and when);
- how much energy has to be stored.

In order to formulate the MPC problem, we next define the cost function associated to the MILP.

A. Cost Function

Microgrid economic optimization is achieved by choosing the decision variables so that a cost functional representing the operating costs is minimized. Therefore, the cost function J includes costs associated to the energy production and start-up and shut-down decisions, along with possible earnings and curtailment penalties. The following cost functional is minimized:

$$J := \sum_{k=0}^{T-1} \sum_{i=1}^{N_g} [C_i^{DG}(P_i(k)) + OM_i\, \delta_i(k) + SU_i(k) + SD_i(k)]$$
$$+ C^{\mathrm{grid}}(k) + \rho_c \sum_{h=1}^{N_c} \beta_h(k) D_h^c(k),$$

where k is the current time instant and T is the length of the prediction horizon. We recall that $C^{\mathrm{grid}}(k)$ can be negative, i.e. energy is sold to the utility grid, representing an earning for the microgrid system. Note that J is a quadratic cost function due to the presence of the quadratic terms $C_i^{DG}(P_i(k))$. Experience has shown that a piecewise affine term, which results in a mixed integer linear program, is more computationally efficient than a quadratic one. We therefore approximate every function $C_i^{DG}(P_i(k))$ with a convex piecewise affine function, which provides very similar results, but can be solved via a mixed integer linear program:

$$C_i^{DG}(P_i) = \max_{w=1,\dots,n} \{S_w P_i + s_w\}, \tag{9}$$

where S_w and s_w are obtained by linearizing the function at n points. We denote by S and s the vectors whose wth row is respectively S_w and s_w.

B. Capacity and terminal constraints

To pose the final MILP optimization problem, additional operational constraints must be met:

$$|\mathbf{F}(k)^{'} u(k) + \mathbf{f}(k)^{'} \hat{w}(k)| \le C_{\max}^b \tag{10a}$$
$$x_{\min}^b \le x^b(k) \le x_{\max}^b \tag{10b}$$
$$P_{i,\min}\, \delta_i(k) \le P_i(k) \le P_{i,\max}\, \delta_i(k) \tag{10c}$$
$$\beta_{h,\min} \le \beta_h(k) \le \beta_{h,\max} \tag{10d}$$
$$|P_i(k+1) - P_i(k)| \le R_{i,\max} \tag{10e}$$

with $i = 1,\dots,N_g$ and $h = 1,\dots,N_c$. The constraints above model the physical bounds on the storage device (inequalities (10a) and (10b)), the power flow limits of the DG units (inequality (10c)), the bounds on controllable loads curtailments (inequality (10d)), and their ramp up and ramp down rates (inequality (10e)).

C. Model predictive control problem

In this section we formulate the MPC optimization problem whose solution yields a trajectory of inputs and states into the future that satisfy the dynamics and constraints of microgrid operations while optimizing some given criteria. In terms of microgrid control, this means that, at the current point in time, an optimal plan is formulated (usually for the 24 hours) based on predictions of the upcoming demand, production from renewable energy units and energy prices. Only the first sample of the input sequence is implemented, and subsequently the horizon is shifted. At the next sampling time, the new state of the system is measured or estimated, and a new optimization problem is solved using this new information. By this receding horizon approach, the new optimal plan can potentially compensate for any disturbance that has meanwhile acted on the system. In order to present the MPC policy, for each time k, we introduce the auxiliary variable $\sigma_i(k)$, which accounts for the ith DG unit generation costs, and the vector $\mathbf{z}(k)$, which collects all the auxiliary variables as follows:

$$\mathbf{z}(k) = \begin{bmatrix} \sigma^{'}(k) & C^g(k) & \mathbf{SU}^{'}(k) & \mathbf{SD}^{'}(k) & z^b(k) \end{bmatrix}^{'} \in \mathbb{R}^{3N_g+2}$$

where $\sigma(k)$, $\mathbf{SU}(k)$, $\mathbf{SD}(k)$ are column vectors containing, respectively, all the $\sigma_i(k)$, the generators start up and start down costs respectively. We denote by $x^b(k+j|k)$, with $j \ge 0$, the state at time step $k+j$ predicted at time k employing the storage model (5). Moreover, we denote by u_k^{T-1} the input sequence $u_k^{T-1} = (u(k),\dots,u(k+T-1))$ designed at time k.

At each time step k, given an initial storage state x_k^b and a final time T, the MPC scheme computes the optimal control sequence u_k^{T-1} solving the following finite-horizon optimal

control problem:

$$J(x_k^b) = \min_{u_k^{T-1}} \sum_{j=0}^{T-1} \left(\mathbf{c_u}(k+j)' u(k+j) + \mathbf{c_z}' \mathbf{z}(k+j) \right),$$

subject to $\qquad\qquad\qquad\qquad\qquad\qquad$ (11)

$\qquad$ storage model (5);

$\qquad$ constraints (6), (7), (8);

$\qquad$ constraints (10);

$\qquad$ $S \cdot P_i(k+j) + s \le \sigma_i(k+j) \; \forall i;$

$\qquad$ $x^b(k|k) = x^b(k),$

where the column vectors $\mathbf{c_z}$ and $\mathbf{c_u}$ are given in the Appendix. We recall that the vector of disturbances profiles, $\hat{w}(k+j)$, is assumed to be known over the prediction horizon, for $j = 0, \ldots, T-1$. According to the receding horizon strategy, only the first element of the optimal sequence $u(k)$ is applied. The optimization problem (11) is repeated at time $k+1$, with the new measured/estimated state $x_{k+1|k+1}^b = x_{k+1}^b$. By doing so, an optimal feedback policy is designed. Note that in the MPC scheme, applied in this preliminary study, the controller makes its control decision by assuming that the predictions are correct (i.e. Certainty Equivalence).

IV. SIMULATION RESULTS

The proposed control strategy is investigated on a typical microgrid and simulation results are presented. This microgrid is in a grid-connected mode and comprises four DG units and photovoltaic panels. An energy storage is included, which is bounded between 10 kWh and 100 kWh and the maximal charge and discharge power are respectively 100 kW and -100 kW. We choose a sampling time of one hour. Simulations are performed over one day. Examples of renewable power production profiles and daily spot prices employed in the optimization routine are depicted respectively in Figure 1 and Figure 2. The microgrid is connected to the utility grid, so power can be bought or sold. The microgrid is allowed to reduce its controllable load preferred level from 10% to 50% in some given times of the day (from 10 am to 16 pm). The demanded power peak can be reduced up to 11%, as shown in Figure 6, leading to a cost reduction of 18%. The Figures 3, 5 and 4 depict, respectively, the energy stored, the power exchanged with the utility grid and the DG unit power production obtained by the MPC control strategy. It is shown that, starting from time 7, the MPC controller decides to turn on the DG units in order to meet the demand at minimum generation costs. In addition, the produced power is meaningfully increased at the times when there is both the highest RES power production and the highest demanded power (about from 10 am to 14 pm). For this reason, the storage device is kept at its maximum capacity during the previous hours and the most convenient amount of energy is bought from the utility grid. The RES power production is utilized either to fulfill the demand or to sell energy to the utility grid.

We used ILOG's CPLEX 12.0 to solve the MILP optimizations, which is an efficient solver based on the branch-and-
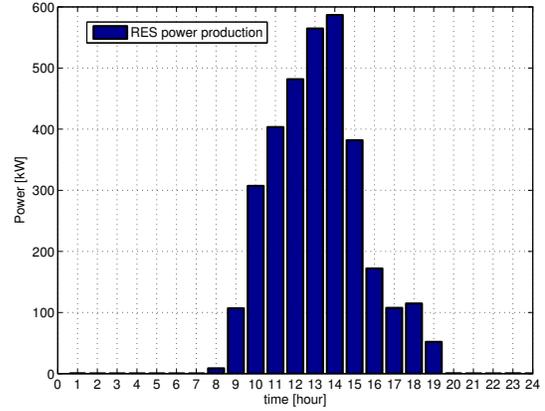

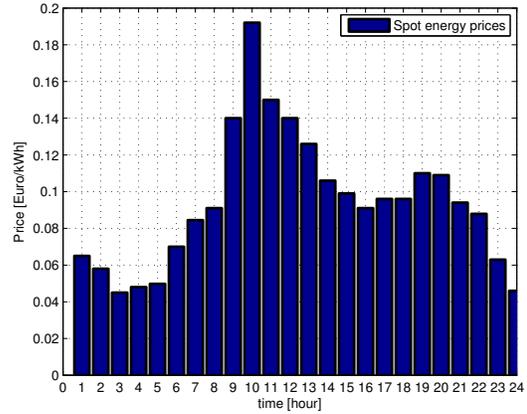Fig. 1. RES power flows over 24 hours.

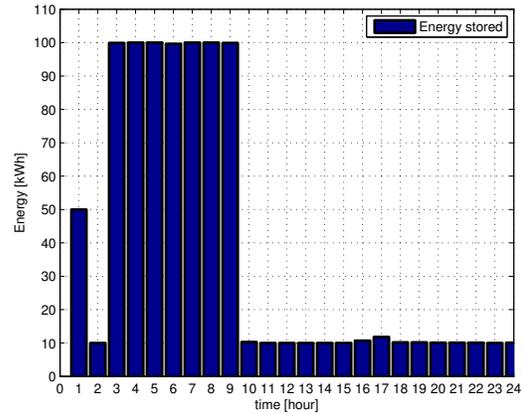Fig. 2. Spot energy prices over 24 hours.

Fig. 3. Stored energy over 24 hours.

bound algorithm [10]. When the branch-and-bound algorithm terminates, the solution is known to be globally optimal. The solution to each MILP problem took at most 6.1 seconds, a time much shorter than the sampling time of one hour. Thus, the computational burden can be affordable.

V. CONCLUSIONS AND FUTURE STEPS

In the paper we proposed a novel mixed integer linear approach on modeling and optimization of microgrids. We bring into account unit commitment, economic dispatch,
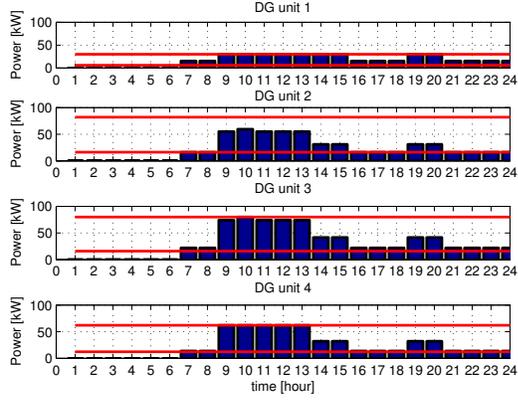
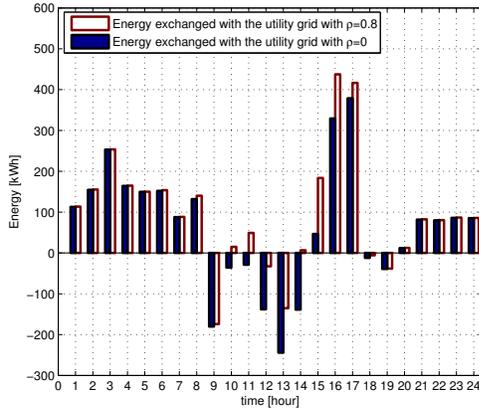Fig. 4. DG units power production over 24 hours ($\rho = 0.8$).

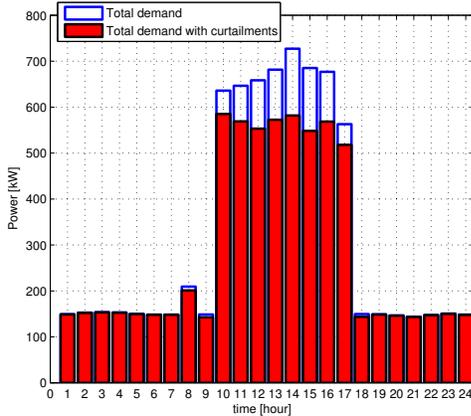Fig. 5. Purchased/sold energy over 24 hours. Notice the difference for different ρ's.

Fig. 6. Curtailments over 24 hours ($\rho = 0.8$).

energy storage, sale and purchase of energy to/from the main grid, curtailment schedule. We assume perfect knowledge of the microgrid state, renewable resources production, future loads, and so on, which is useful to solve the optimization problem. Further, to cope with inevitable disturbances and forecast errors, we embed this into an MPC framework. Future work will focus on applying more realistic approaches to microgrid optimization, including state estimation and uncertainty modeling. Moreover we are currently studying

realistic and less conservative approaches which ensure the stability and the feasibility of proposed MPC strategy.

VI. APPENDIX

$$\mathbf{E_1^{b}}' = \begin{bmatrix} C^b & -(C^b + \varepsilon) & C^b & C^b & -C^b & -C^b \end{bmatrix}$$

$$\mathbf{E_2^{b}}' = \mathbf{E_2^{g}}' = \begin{bmatrix} 0 & 0 & 1 & -1 & 1 & -1 \end{bmatrix}$$

$$\mathbf{E_3^{b}}' = \begin{bmatrix} 1 & -1 & 1 & -1 & 0 & 0 \end{bmatrix}$$

$$\mathbf{E_4^{b}}' = \begin{bmatrix} C^b & -\varepsilon & C^b & C^b & 0 & 0 \end{bmatrix}$$

$$\mathbf{E_1^{g}}' = \begin{bmatrix} T^g & -(T^g + \varepsilon) & T^g & T^g & -T^g & -T^g \end{bmatrix}$$

$$\mathbf{E_3^{g}}'(k) = \begin{bmatrix} 1 & -1 & c^P(k) & -c^P(k) & c^S(k) & -c^S(k) \end{bmatrix}$$

$$\mathbf{E_4^{g}}' = \begin{bmatrix} T^g & -\varepsilon & T^g & T^g & 0 & 0 \end{bmatrix}$$

where ε is a small tolerance (typically the machine precision) needed to transform a strict inequality constraint into a nonstrict inequality, since in MILP solving algorithm only nonstrict inequalities can be handled [9].

$$\mathbf{F}'(\mathbf{k}) = [\underbrace{1 \ldots 1}_{Ng} \ D_1^c(k) \ \ldots \ D_{Nc}^c(k) \ \underbrace{0 \ldots 0}_{Ng}]$$

$$\mathbf{f}'(k) = [1 \ -D_1(k) \ \ldots \ -D_{Nl}(k) \ -D_1^c(k) \ \ldots \ -D_{Nc}^c(k)]$$

$$\mathbf{c_z}' = [\underbrace{1 \ldots 1}_{Ng} \ 1 \ \underbrace{1 \ldots 1}_{Ng} \ 0],$$

$$\mathbf{c_u}(k)' = [\underbrace{0 \ldots 0}_{Ng} \ OM_1 \ldots OM_{Nl} \ 1$$
$$\beta_1(k)D_1^c(k) \ldots \beta_{Nc}(k)D_{Nc}^c(k)].$$

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