A Mixed Integer Linear Formulation for Microgrid Economic Scheduling

Alessandra Parisio and Luigi Glielmo
Department of Engineering
Università degli Studi del Sannio
Piazza Roma 21, 82100, Benevento
Email: aparisio@unisannio.it

***Abstract*—Microgrids are subsystems of the distribution grid which comprises small generation capacities, storage devices and controllable loads, which can operate either connected or isolated from the utility grid. This paper studies the microgrid economic scheduling, i.e. the problem of optimize microgrid operations to fulfil a time-varying energy demand and operational constraints while minimizing the costs of internal production and imported energy from the utility grid. The problem is posed as a mixed-integer linear programming model. The key difference in the proposed modeling approach is that no complex heuristics or decompositions are used; the full model is formulated and solved in an efficient way by using commercial solvers. This leads to significant improvements in schedule quality and in computational burden. A case study of a typical microgrid is investigated: simulation results show the feasibility and the effectiveness of the proposed approach.**

I. Introduction

The need to satisfy in sustainable ways the increasing energy demand requires *active* energy distribution networks, i.e. distribution networks with the possibility of bidirectional power flows controlling a combination of Distributed Energy Resources (DERs), such as distributed generators and renewable energy devices. Hence, new energy management systems are needed, able to optimally schedule the distributed generation in the distribution network. In this scenario, the microgrid concept is a promising approach. It is an integrated energy system consisting of interconnected loads and DERs which can operate in parallel with the grid or in an intentional island mode [1], [2]. A typical microgrid comprises: storage units; Distributed Generators (DGs), which are dispatchable units; Renewable Energy Resources (RESs), which are non controllable devices; and controllable loads, which can be curtailed (shed) when it is more convenient. In addition a microgrid can purchase and sell power to and from its energy suppliers. The optimization of the microgrid operations is extremely important in order to cost-efficiently manage its energy resources [2], [3]. In this paper we tackle the optimal scheduling of microgrid operations. This problem aims at minimizing the production costs of the local generators and the exchange with the utility grid subject to market conditions, while satisfying a predicted load demand of a certain period (typically one day) and complex operational constraints, such as the energy balance, and controllable generators minimum operation time and minimum stop time. A complete formulation of microgrid economic scheduling problem includes modeling of storage, demand side policies for controllable loads (Demand Side Management, DSM), power exchange with the utility grid. The problem is generally formulated as a Mixed Integer Nonlinear Problem (MINLP) for which there is no exact solution technique. Namely, microgrid modeling needs both continuous (such as storage output) and discrete (such as on/off states of DGs and DSM-controlled loads) decision variables, which causes the solution space of the corresponding optimization problem to be nonconvex, so that classical mathematical programming techniques cannot be directly applied. Due to the problem complexity and because of the large economic benefits that could result from its improved solution, considerable attention is being devoted to development of better optimization algorithms and suitable modeling frameworks. Moreover studies have suggested that microgrids can achieve high performance through:(*i*) deployment of demand response; (*ii*) optimal use of storage devices in order to compensate the physical imbalances; (*iii*) applying optimal instead of heuristic-based approaches [4]–[7]. The proposed approaches are typically either computationally intensive and not suitable for online applications, or can produce suboptimal solutions. Most proposed solution methods utilize heuristic-based methods using priority list, dynamic programming, Lagrangian relaxation, genetic algorithms, particle swarm optimization, usually in a deterministic setting [8]–[10]. In a stochastic framework, the authors in [11] propose an optimization algorithm based on dynamic programming. In the aforementioned work, the optimization problem stays nonlinear and important features such as minimum up and down times and demand side programs are neglected. Therefore it is necessary to find a tractable formulation of the microgrid operation optimization problem which includes the specific key features of a microgrid. In this paper we present a mixed integer linear formulation of the microgrid economic scheduling. By applying well known linear approximation techniques of nonlinear cost functions and formulating complex operative constrains as linear constraints, the problem can be solved very efficiently by standard algorithms [12]–[14]. Moreover, the possibility to implement load curtailment programs is also included. Our main contributions are: (*i*) the development of a model of the overall microgrid system adopting a formalized modeling approach; (*ii*) the formulation of the microgrid scheduling problem so as it is suitable to be used in online optimization schemes;(*iii*) the presentation of preliminary simulation results showing the effectiveness of the proposed optimization routine.

The paper is organized as follows: the microgrid system and the modeling approach are described in Section II; the operation optimization is then described in Section III; finally, in Section IV some simulation results are discussed.

A. Nomenclature

The forecasts, the parameters and the decision variables used in the proposed formulation are described respectively in Tables I, II and III.

TABLE I
PARAMETERS

Parameters	Description
N_g	number of DG units
N_l	number of critical loads
N_c	number of controllable loads
$C^{DG}(P)$	fuel consumption cost curve of a DG unit
a_1, a_2, a_3	cost coefficients of $C^{DG}(P)$ [€/(kWh)^2, €/kWh, €]
OM	operating and maintenance cost of a DG unit [€/kWh]
$R_{\max}$	ramp up limit of a DG unit [kW/h]
T^{up}	minimum up time of a DG unit [h]
T^{down}	minimum down time of a DG unit [h]
x^{sb}	storage ‘physiological’ energy loss per time step (hour) [kWh]
$x^b_{\min}$, $x^b_{\max}$	minimum, maximum energy level of the storage unit [kWh]
$C^b_{\max}$	storage power limit [kW]
T^g	maximum interconnection power flow limit (at the point of common coupling) [kW]
$P_{\min}$, $P_{\max}$	minimum, maximum power level of a DG unit [kW]
η^c, η^d	storage charging, discharging ”efficiencies”
$\beta_{\min}$, $\beta_{\max}$	minimum, maximum allowed curtailment of a controllable load
c^{SU}, c^{SD}	start-up, start-down costs of a DG unit [€]
D^c	preferred power level of a controllable load [kW]
ρ_c	penalty weight on curtailments

We assume a quadratic fuel consumption cost for a DG unit of the form $C^{DG}(P) = a_1P^2 + a_2P + a_3$.

TABLE II
FORECASTS

Forecasts	Description
P^{res}	sum of power production from RES [kW]
D	power level required from a non controllable load [kW]
c^P, c^S	purchasing, selling energy prices [€/kWh]

TABLE III
DECISION AND LOGICAL VARIABLES

Variables	Description
δ	off(0)/on(1) state of a DG unit
δ^b	discharging(0)/charging(1) mode of a storage unit
δ^g	exporting(0)/importing(1) mode to/from the utility grid
P	power level of a DG unit [kW]
P^b	power exchanged (positive for charging) with the storage unit [kW]
P^g	importing(positive)/exporting(negative) power level from/to the utility grid [kW]
x^b	stored energy level [kWh]
β	curtailed power percentage

II. SYSTEM DESCRIPTION AND MODELING

Here we briefly describe the key features of the microgrid architecture considered in this paper and associate a possible modeling set up with the goal of maintaining the problem tractable. When the microgrid is in the grid-connected mode, it can purchase and sell energy from the utility grid. The microgrid produces the electricity using controllable distributed generators and renewable energy resources, and energy can be stored in a storage device. The energy demand comes from both critical and controllable loads. The optimal use of the storage unit and the controllable loads can help to keep the energy balance, in particular during the islanded mode. The microgrid system comprises continuous time-driven dynamics of the energy flows and storage units, and event-driven on/off controllers.

We point out what follows:

- heat recovery capabilities and reactive power are not considered in the microgrid modeling and problem formulation to limit its complexity. Yet we are aware of their importance and their incorporation into the proposed control framework is under current study.
- due to constant sampling time $\Delta T = t_{k+1} - t_k$, there exists a constant ratio between energy and power at each interval.

A. Storage Dynamics

We consider the following discrete time model of a storage unit:

$$x^b(k+1) = x(k)^b + \eta P^b(k) - x^{sb}, \tag{1}$$

where

$$\eta = \begin{cases} \eta^c, & \text{if } P^b(k) > 0 \text{ (charging mode)} \\ \eta^d, & \text{otherwise (discharging mode).} \end{cases} \tag{2}$$

where typically $\eta^c < 1$ and $\eta^1 = \frac{1}{\eta^c}$. We denote by $x^b(k)$ the level of the energy stored at time k (divided by ΔT) and by $P^b(k)$ the power exchanged with the storing device at time k. The charging and discharging ”efficiencies” account for the losses and x_{sb} denotes a constant stored energy degradation in the sampling interval. If the power exchanged at time k, $P^b(k)$, is greater than zero, this will be charging the storage device, otherwise the storage device will be discharged. By using the standard approach described in [15], we introduce a binary variable $\delta^b(k)$ and an auxiliary variable $z^b(k) = \delta^b(k)P^b(k)$ to model the logical conditions provided in Section 4 such as:

$$P^b(k) > 0 \Longleftrightarrow \delta^b(k) = 1$$

and

$$x^b(k+1) = \begin{cases} x^b(k) + \eta^c P^b(k) - x^{sb}, & \text{if } \delta^b(k) = 1, \\ x^b(k) + \eta^d P^b(k) - x^{sb}, & \text{otherwise.} \end{cases}$$

Then we express the ‘if … then’ conditions as mixed integer linear inequalities. By collecting such inequalities we can rewrite the storage dynamics and the corresponding constraints in the following compact form (the interested reader is referred to [15] for guiding details):

$$x^b(k+1) = x^b(k) + (\eta^c - \eta^d)z^b(k) + \eta^d P^b(k) - x^{sb}, \\ \text{subject to} \quad \mathbf{E_1}^b \delta^b(k) + \mathbf{E_2}^b z^b(k) \le \mathbf{E_3}^b P^b(k) + \mathbf{E_4}^b, \tag{3}$$

where the column vectors $\mathbf{E_1}^b, \mathbf{E_2}^b, \mathbf{E_3}^b, \mathbf{E_4}^b$ are provided in the Appendix VI.

The balance between energy production and consumption must be met at each time k, so the following equality constraint is imposed:

$$\begin{aligned} P^b(k) \;\; = & \sum_{i=1}^{N_g} P_i(k) + P^{\text{res}}(k) + P^g(k) \\ & - \sum_{j=1}^{N_l} D_j(k) - \sum_{h=1}^{N_c} [1 - \beta_h(k)] D_h^c(k). \end{aligned} \tag{4}$$

If we collect all the decision variables in the vector $u(k)$ and all the known disturbances (obtained by forecasts) in the vector $\hat{w}(k)$, we can express the storage level as an affine function by substituting $P^b(k)$ in (3) as follows:

$$\begin{aligned} x^b(k+1) \;\; = \;\; & x^b(k) + (\eta^c - \eta^d) z^b(k) \\ & + \eta^d \left[\mathbf{F}'(k) u(k) + \mathbf{f}'(k) \hat{w}(k)\right] - x^{sb} \end{aligned} \tag{5}$$

with

$$u(k) = \left[\mathbf{P}'(k) \;\; P^g(k) \;\; \beta'(k) \;\; \delta'(k)\right]' \in \mathbb{R}^{N_u} \times \{0,1\}^{N_g},$$

$$\hat{w}(k) = \left[P^{\text{res}}(k) \;\; \mathbf{D}'(k) \;\; \mathbf{D^c}'(k)\right]' \in \mathbb{R}^{N_w},$$

where $N_u = N_g + 1 + N_c$, $N_w = 1 + N_l + N_c$, $\mathbf{P}(k)$, $\delta(k)$, $\mathbf{D}(k)$, $\mathbf{D^c}(k)$ and $\beta(k)$ are column vectors containing, respectively, all the power levels, the generators off/on states, the critical demands, the controllable preferred power levels and the curtailments. We remark that the vector $u(k)$ collects both the continuous-value and the binary decision variables, and the vector $\hat{w}(k)$ collects all the known disturbances (obtained by forecasts). The vectors $\mathbf{F}'(k)$ and $\mathbf{f}'(k)$ are provided in the Appendix VI.

B. Interaction with the utility grid

When it is grid-connected, the microgrid can sell and purchase energy from the utility grid. If the power exchanged with the utility grid at time k, $P^g(k)$, has positive sign, energy is purchased from the utility grid within the sampling time; otherwise energy is sold to the utility grid.

By following the same procedure outlined above, we introduce a binary variable $\delta^g(k)$ and an auxiliary variables $C^g(k)$ to model the possibility either to purchase or to sell energy from/to the utility grid. For the new variables, the following logical statements must hold:

$$P^g(k) > 0 \Longleftrightarrow \delta^g(k) = 1$$

and

$$C^g(k) = \begin{cases} c^P(k) P^g(k) & \text{if } \delta^g(k) = 1, \\ c^S(k) P^g(k) & \text{otherwise.} \end{cases}$$

Again, we express the 'if … then' conditions as mixed integer linear inequalities. Then, the purchasing/selling microgrid behavior can be expressed by the following mixed integer linear inequalities in a compact form:

$$\mathbf{E_1}^g \delta^g(k) + \mathbf{E_2}^g C^g(k) \le \mathbf{E_3}^g(k) P^g(k) + \mathbf{E_4}^g. \tag{6}$$

The column vectors $\mathbf{E_1}^g, \mathbf{E_2}^g, \mathbf{E_3}^g(k), \mathbf{E_4}^g$ are provided in the Appendix VI. The matrix $\mathbf{E_3}^g(k)$ is generally time-varying due to the time varying energy prices. We recall that the interaction with the utility grid is allowed only when the microgrid is in the grid-connected mode.

C. Generator operating conditions

The operating constraints, at each sampling time k, on the minimum amount of time for which a controllable generation unit must be kept on/off (minimum up/down times) can be expressed by the following mixed integer linear inequalities without resorting to any additional variable:

$$\begin{aligned} & \delta_i(k) \ge \delta_i(k - \tau_{\text{up}} - 1) - \delta_i(k - \tau_{\text{up}} - 2), \\ & 1 - \delta_i(k) \ge \delta_i(k - \tau_{\text{down}} - 2) - \delta_i(k - \tau_{\text{down}} - 1), \end{aligned} \tag{7}$$

with $i = 1, \dots, N_g$, $\tau_{\text{up}} = 0, \dots, \min(T_i^{\text{up}} - 1, k - T_i^{\text{up}} + 2)$ and $\tau_{\text{down}} = 0, \dots, \min(T_i^{\text{up}} - 1, k - T_i^{\text{up}} + 2)$.

We also model the DG unit start up and shut down behavior in order to account for the corresponding costs. For this reason, two auxiliary variables, $SU_i(k)$ and $SD_i(k)$ are introduced, representing respectively the start up and the shut down cost for the i^{th} DG generation unit at time k. These auxiliary variables must satisfy the following mixed integer linear constraints:

$$\begin{aligned} & SU_i(k) \ge c_i^{SU}(k)[\delta_i(k) - \delta_i(k-1)], \\ & SD_i(k) \ge c_i^{SD}(k)[\delta_i(k-1) - \delta_i(k)], \\ & SU_i(k) \ge 0, \\ & SD_i(k) \ge 0, \end{aligned} \tag{8}$$

with $i = 1, \dots, N_g$.

D. Loads

We consider two types of loads:

- *critical loads*, i.e. demand levels related to essential processes that must be always met;
- *controllable loads*, i.e. loads that can be reduced or shed during supply constraints or emergency situations (e.g., standby devices, day-time lighting).

In demand response programs the customers specify level of curtailment of the controllable loads. The controllable loads have a preferred level, but their magnitude is flexible so that the demand level can be lowered when it is convenient or necessary (e.g., in islanded mode). This leads to users' discomfort, hence a certain cost is associated with the load curtailment/shedding (a penalty for the microgrid).

We define a continuous-valued variable, $0 \le \beta(k) \le 1$, associated to each controllable load c and to each sampling time k. This variable represents the percentage of preferred power level to be curtailed at time k in order to keep the microgrid operations feasible (e.g., in islanded mode) or more economically convenient. If no curtailment is allowed at a certain time $\hat{k}$, an equality constraint can be set, $\beta(\hat{k}) = 0$.

III. Problem Formulation

In this section we define the microgrid economic scheduling problem. At every time step, the microgrid scheduler must take high level decisions about:

- when should each generation unit be started and stopped (Unit Commitment);
- how much should each unit generate to meet this load at minimum cost (Economic Dispatch);
- when should the storage device be charged or discharged;
- when and how much energy should be purchased or sold to the utility grid (when the microgrid is in the grid-connected mode);
- curtailment schedule (which controllable loads must be shed/curtailed and when);
- how much energy has to be stored.

In order to formulate the microgrid scheduling problem, we next define the cost function associated with the MILP.

A. Cost Function

Microgrid economic optimization is achieved by designing the decision variables so that a cost functional representing the operating costs is minimized. Therefore, the cost function, J, includes costs associated with energy production and start-up and shut-down decisions, along with possible earnings and curtailment penalties. The following cost functional is minimized:

$$J := \sum_{k=0}^{T-1} \sum_{i=1}^{N_g} [C_i^{DG}(P_i(k)) + OM_i\, \delta_i(k) + SU_i(k) + SD_i(k)] + C^{\text{grid}}(k) + \rho_c \sum_{h=1}^{N_c} \beta_h(k) D_h^c(k),$$

where k is the current time instant and T is the length of the prediction horizon. We recall that $C^{\text{grid}}(k)$ can be negative, i.e. energy is sold to the utility grid, representing an earning for the microgrid system. Note that J is a quadratic cost function due to the presence of the quadratic terms $C_i^{DG}(P_i(k))$. Experience has shown that a piecewise affine term, which results in a mixed integer linear program, is more computationally efficient than a quadratic one. We therefore approximate every function $C_i^{DG}(P_i(k))$ with a convex piecewise affine function, which provides very similar results, but can be solved via a mixed integer linear program.

B. Capacity and terminal constraints

To pose the final MILP optimization problem, additional operational constraints must be met:

$$|\mathbf{F}(k)' u(k) + \mathbf{f}(k)' \hat{w}(k)| \leq C_{\max}^b \tag{9a}$$

$$x_{\min}^b \leq x^b(k) \leq x_{\max}^b \tag{9b}$$

$$P_{i,\min}\, \delta_i(k) \leq P_i(k) \leq P_{i,\max}\, \delta_i(k) \tag{9c}$$

$$\beta_{h,\min} \leq \beta_h(k) \leq \beta_{h,\max} \tag{9d}$$

$$|P_i(k+1) - P_i(k)| \leq R_{i,\max} \tag{9e}$$

with $i = 1, \ldots, N_g$ and $h = 1, \ldots, N_c$. The constraints above model the physical bounds on the storage device (inequalities (9a) and (9b)), the power flow limits of the DG units (inequality (9c)), the bounds on controllable loads curtailments (inequality (9d)), and their ramp up and ramp down rates (inequality (9e)). Moreover, the stored energy levels both at the beginning and at the end of the planning period are usually assumed to be equal to the 50% of the maximum storage power limit. Therefore, the following terminal equality constraint must be enforced:

$$x^b(T) = x^b(0). \tag{10}$$

C. Microgrid scheduling problem

The microgrid economic scheduling problem can be stated as the following finite-horizon optimal control problem:

$$\begin{aligned} &\min J \\ &\text{subject to} \\ &\qquad \text{storage model (5);} \\ &\qquad \text{constraints (6), (7), (8);} \\ &\qquad \text{constraints (9).} \end{aligned} \tag{11}$$

At the current point in time, an optimal sequence of decisions is formulated (usually for the 24 hours) based on predictions of the upcoming demand, production from renewable energy units and energy prices.

IV. Simulation results

The proposed control strategy is investigated in simulation on a typical microgrid. This microgrid is in a grid-connected mode and comprises four DG units and photovoltaic panels. An energy storage is included, which is bounded between 10 kWh and 100 kWh and the maximal charge and discharge power are respectively 100 kW and -100 kW. Tables IV and V describe the DG units parameters. We choose a sampling time of one hour. Simulations are performed over one day. Examples of daily spot prices and renewable power production profiles employed in the optimization routine are depicted respectively in Figure 2 and Figure 1. The microgrid

TABLE IV
Generator parameters

DG unit	P_{min}	P_{max}	a_1	a_2	a_3
Unit 1	6	30	0.00637	0.248	4.011
Unit 2	16.4	82	0.004209	0.2304	3.428
Unit 3	16	80	0.00209	0.2254	3.428
Unit 4	12.3	62	0.003026	0.2278	5.722

TABLE V
Generator parameters

DG unit	T^{up}	T^{down}	R_{max}	OM	c^{SU}	c^{SD}
Unit 1	4	4	30	0.09	4	4
Unit 2	2	2	60	0.05	2	2
Unit 3	4	4	60	0.09	3	3
Unit 4	2	2	50	0.08	3	3

is connected to the utility grid, so energy can be bought or sold. We consider the general case of a microgrid that disposes of a controllable load which may be reduced up to a limit without having to feed it afterwards. The upper and lower bounds on the allowed curtailments can be set by bilateral contracts. In particular, the microgrid is requested to reduce its controllable load preferred level in some given times of the day (from 10 am to 16 pm); the reduction percentage is upper bounded by a value varying from 10% to 50%. Curtailments are usually penalized since they lead to user discomfort; so they are not performed unless strictly convenient or necessary. The penalty factor on the curtailments, ρ, is set to 0.7. The Figures 4, 5 and 3 depict, respectively, the energy stored, the exchanged power with the utility grid and the DG unit power production obtain by the MILP optimization routine. It is shown that, starting from time 7, the MILP scheduler decides to turn on the DG units in order to meet the demand at minimum generation costs. In addition, the produced power is meaningfully increased at the times when there is both the highest RES power production and the highest demanded power (about from 10 am to 14 pm). For this reason, the storage device is kept at its maximum level during the previous hours and the most convenient amount of energy is bought from the utility grid. The RES power production is utilized either to charge the storage, or to fulfil the demand or to sell energy to the utility grid. For instance, at time 9, the scheduler deems that the best decision to make is to further increase the power production of the DG units and to utilize the RES power production both to fulfill the demand and to sell energy to the utility grid. By doing this, the storage level can be kept at its maximum in order to support the upcoming higher demanded power. At hour 13, the DG units still produce a significant amount of power. The power from the controllable and RES units and the power discharged from the storage device (which is kept at its minimum level this time), can satisfy the demand; the power surplus can be then sold to the utility grid. Since the penalty with ρ is set to 0.7, the curtailments are not performed all the times when they are allowed, yielding only the 4.2% total demand peak reduction. The demanded power peak can be further reduced by decreasing the parameter ρ; this comes at a price of a lower user comfort.

We use ILOG's CPLEX 11.0 [16] to solve the MILP problems, which are known to be are NP-complete. CPLEX is an efficient solver based on the branch-and-bound techniques [12], [17]. The main advantage of the branch and bound method is that, when it terminates, the solution is known to be globally optimal. The proposed MILP optimization problem is suitable for online applications; namely, in the case study reported above, the MILP problem is solved in $4.24s$, a time much shorter than the sampling time of one hour. This can also mean that the sampling time could be reduced at the cost of a negligible computational bigger effort.

V. Conclusions and Future Steps

In the paper we proposed a novel mixed integer linear approach on modeling and optimization of microgrids. We bring into account unit commitment, economic dispatch, energy storage, sale and purchase of energy to/from the main grid, curtailment schedule. We assume perfect knowledge of the microgrid state, renewable resources production, future loads, and so on, which is useful to solve the optimization problem. The authors will apply the algorithms here described to an experimental laboratory plant and may be able to include experimental data in the final version of the paper. We are currently studying realistic and effective stochastic approaches to cope with inherent uncertainty due to RES production, energy demand and prices. Moreover, future studies will include the reactive power management.

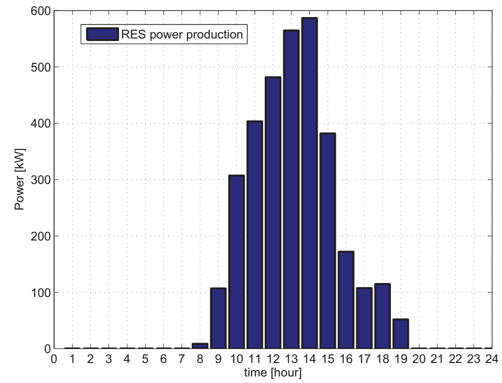

Fig. 1. RES power flows over 24 hours.

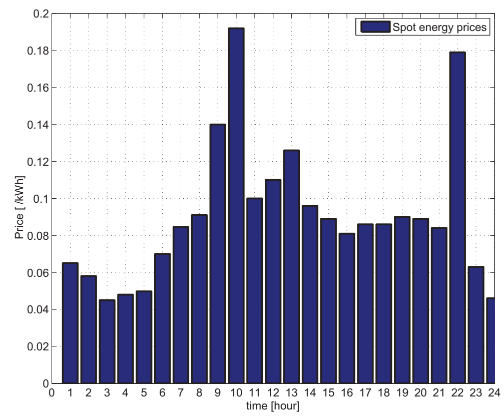

Fig. 2. Spot energy prices over 24 hours.

VI. Appendix

$$\mathbf{E_1^{b}}' = \begin{bmatrix} C^b & -(C^b+\varepsilon) & C^b & C^b & -C^b & -C^b \end{bmatrix}$$

$$\mathbf{E_2^{b}}' = \mathbf{E_2^{g}}' = \begin{bmatrix} 0 & 0 & 1 & -1 & 1 & -1 \end{bmatrix}$$

$$\mathbf{E_3^{b}}' = \begin{bmatrix} 1 & -1 & 1 & -1 & 0 & 0 \end{bmatrix}$$

$$\mathbf{E_4^{b}}' = \begin{bmatrix} C^b & -\varepsilon & C^b & C^b & 0 & 0 \end{bmatrix}$$

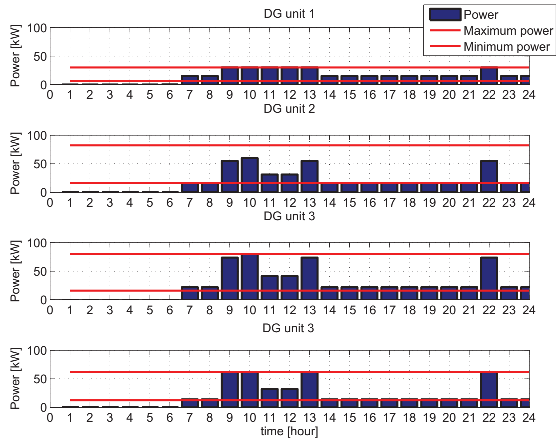

Fig. 3. DG units power production over 24 hours.

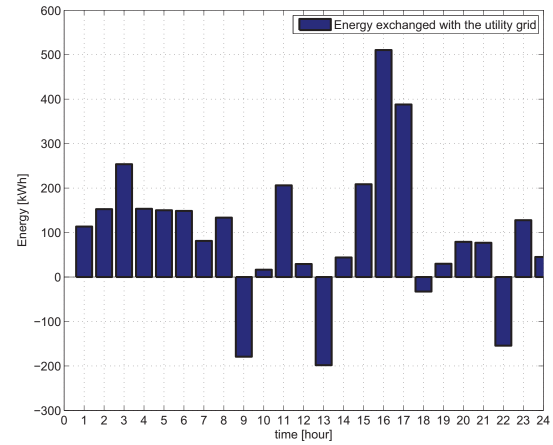

Fig. 5. Purchased and sold energy over 24 hours.

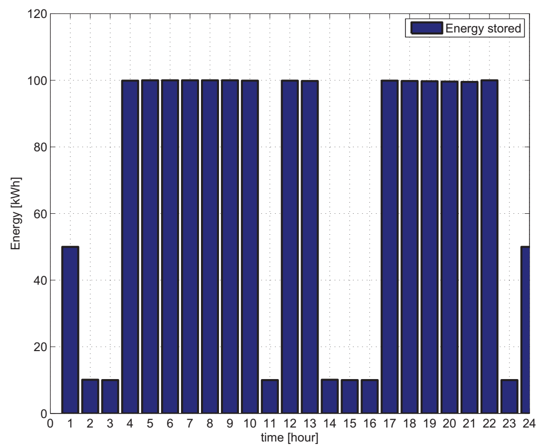

Fig. 4. Stored energy over 24 hours.

$$\mathbf{E_1^{g}}' = \begin{bmatrix} T^g & -(T^g+\varepsilon) & T^g & T^g & -T^g & -T^g \end{bmatrix}$$

$$\mathbf{E_3^{g}}'(k) = \begin{bmatrix} 1 & -1 & c^P(k) & -c^P(k) & c^S(k) & -c^S(k) \end{bmatrix}$$

$$\mathbf{E_4^{g}}' = \begin{bmatrix} T^g & -\varepsilon & T^g & T^g & 0 & 0 \end{bmatrix}$$

where ε is a small tolerance (typically the machine precision) needed to transform a strict inequality constraint into a nonstrict inequality, since in MILP solving algorithm only nonstrict inequalities can be handled [15].

$$\mathbf{F}'(\mathbf{k}) = \begin{bmatrix} \underbrace{1 \ldots 1}_{Ng} & D_1^c(k) \ldots D_{Nc}^c(k) & \underbrace{0 \ldots 0}_{Ng} \end{bmatrix}$$

$$\mathbf{f}'(k) = \begin{bmatrix} 1 & -D_1(k) \ldots -D_{Nl}(k) & -D_1^c(k) \ldots -D_{Nc}^c(k) \end{bmatrix}$$

REFERENCES

[1] R. Lasseter and P. Piagi, "Microgrid: a conceptual solution," in *IEEE Annual Power Electron Specialists Conference*, 2004, pp. 4285–4290.

[2] N. Hatziargyriou, H. Asano, R. Iravani, and C. Marnay, "Microgrids," *IEEE Power & Energy Magazine*, 2007.

[3] "Strategic deployment document for europes electricity networks of the future," 2008, http://www.smartgrids.eu/.

[4] R. Firestone and C. Marnay, "Energy manager design for microgrids," LBNL Paper LBNL-54447, Lawrence Berkeley National Laboratory, Tech. Rep., 2005.

[5] A. Siddiqui, C. Marnay, O. Bailey, and K. LaCommare, "Optimal selection of on-site power generation with combined heat and power applications," *International Journal of Distributed Energy Resources*, vol. 1, no. 1, pp. 33–62, 2005.

[6] G. Pepermans, J. Driesen, D. Haeseldonckx, R. Belmans, and W. D'haeseleer, "Distributed generation: Definition, benefits and issues," *Energy Policy*, vol. 33, no. 6, pp. 787–798, 2005.

[7] A. Siddiqui and C. Marnay, "Operation of distributed generation under stochastic prices," *Pacific Journal of Optimization*, vol. 3, no. 3, pp. 439–458, 2007.

[8] A. Siddiqui, C. Marnay, R. Firestone, and N. Zhou, "Distributed generation with heat recovery and storage," *Journal of Energy Engineering*, vol. 133, no. 3, pp. 181–210, 2007.

[9] F. Mohamed, "Microgrid modelling and online management," Ph.D. dissertation, Helsinki University of Technology (Espoo, Finland), 2008.

[10] A. Milo, H. Gaztanaga, I. Etxeberria-Otadui, E. Bilbao, and P. Rodriguez, "Optimization of an experimental hybrid microgrid operation: Reliability and economic issues," in *IEEE PowerTech Conference 2009*, Bucharest, Romania, 2009, pp. 1–6.

[11] L. Costa and G. Kariniotakis, "Stochastic dynamic programming model for optimal use of local energy resources in a market environment," in *IEEE Power Tech*, 2007, pp. 449–454.

[12] C. Floudas, *Nonlinear and Mixed-Integer Programming Fundamentals and Applications*. Oxford, UK: Oxford University Press, 1995.

[13] E. Sontag, "Verification and control," in *Hybrid Systems III*, ser. Lecture Notes in Computer Science, in R. Alur, T. Henzinger, and e. E.D. Sontag, Eds., vol. 1066. Springer-Verlag, 1996, pp. 436–448.

[14] G. Nemhauser and L. Wolsey, *Integer and Combinatorial Optimization*. Wiley, 1988.

[15] A. Bemporad and M. Morari, "Control of systems integrating logic, dynamics, and constraints," *Automatica*, vol. 35, no. 3, 1999.

[16] ILOG, *CPLEX 12.0 Users Manual*.

[17] D. Bertsimas and J. Tsitsiklis, *Introduction to Linear Optimization*. Belmont MA: Athena Scientific, 1997.

[18] A. Richard and J. How, 8 2005.